UNIT-4 YVNR

ELECTROMAGNETIC WAVES-I

→ The waves are the means of transporting energy (or) information from source to destination.

→ The waves consisting electric and magnetic fields are called electromagnetic waves. The wave is a function of time and space.

→ The waves are travel with high velocity (velocity of light in free space) and they radiate outwards from a source in all directions.

→ Typical examples of EM waves are Radio waves, TV signals, Radar beams, light rays etc.

→ In this chapter, we will discuss analysis of the maxwell's equations to derive equations of EM waves in following media such as

1. Free space ($\sigma = 0$, $\epsilon = \epsilon_0$, $\mu = \mu_0$)
2. lossless dielectrics ($\sigma = 0$, $\epsilon = \epsilon_r \epsilon_0$, $\mu = \mu_r \mu_0$, $\frac{\sigma}{\omega\epsilon} << 1$)
3. lossy dielectrics ($\sigma \neq 0$, $\epsilon = \epsilon_r \epsilon_0$, $\mu = \mu_r \mu_0$)
4. Good conductors ($\sigma = \infty$, $\epsilon = \epsilon_0$, $\mu = \mu_r \mu_0$, $\frac{\sigma}{\omega\epsilon} >> 1$)

General wave equations :

Electromagnetic waves can be obtained from maxwell's equations

$D = \epsilon E$

$B = \mu H$

$$\nabla \cdot D = \rho_v \Rightarrow \nabla \cdot E = \frac{\rho_v}{\epsilon} \longrightarrow (1)$$

$$\nabla \cdot B = 0 \Rightarrow \nabla \cdot H = 0 \longrightarrow (2)$$

$$\nabla \times E = -\frac{\partial B}{dt} \Rightarrow \nabla \times E = -\mu \frac{\partial E}{dt} \longrightarrow (3)$$

$$\nabla \times H = J + \frac{\partial D}{dt} \Rightarrow \nabla \times H = \sigma . E + \epsilon \frac{\partial E}{dt} \longrightarrow (4)$$

→ Apply curl operation to eq (3)

$$\nabla \times \nabla \times E = \nabla \times \left(-\mu \frac{dH}{dt}\right)$$

$$\nabla(\nabla \cdot E) - (\nabla \cdot \nabla)E = -\mu \frac{d}{dt}(\nabla \times H)$$

$$\nabla\left(\frac{\rho_v}{\epsilon}\right) - \nabla^2 E = -\mu \frac{d}{dt}\left(\sigma E + \epsilon \frac{dE}{dt}\right)$$

$$\nabla\left(\frac{\rho_v}{\epsilon}\right) - \nabla^2 E = -\mu\sigma \frac{dE}{dt} - \mu\epsilon \frac{d^2E}{dt^2}$$

$$\boxed{\therefore \nabla^2 E = \mu\sigma \frac{dE}{dt} + \mu\epsilon \frac{d^2E}{dt^2} + \nabla\left(\frac{\rho_v}{\epsilon}\right)} \longrightarrow (5)$$

→ Apply curl operation to eq (4)

$$\nabla \times \nabla \times H = \nabla \times \left(\sigma E + \epsilon \frac{dE}{dt}\right)$$

$$\nabla(\nabla \cdot H) - (\nabla \cdot \nabla)H = \sigma(\nabla \times E) + \epsilon \frac{d}{dt}(\nabla \times E)$$

$$0 - \nabla^2 H = -\sigma\mu \frac{dH}{dt} + \epsilon \frac{d}{dt}\left(-\mu \frac{dH}{dt}\right)$$

$$-\nabla^2 H = -\sigma\mu \frac{dH}{dt} - \epsilon\mu \frac{d^2H}{dt^2}$$

$$\boxed{\therefore \nabla^2 H = \mu\sigma \frac{dH}{dt} + \mu\epsilon \frac{d^2H}{dt^2}} \longrightarrow (6)$$

→ In general mediums (or) charge free regions, $\rho_v = 0$, then

$$(5) \Rightarrow \nabla^2 E = \mu\sigma \frac{dE}{dt} + \mu\epsilon \frac{d^2E}{dt^2} \longrightarrow (7)$$

$$(6) \Rightarrow \nabla^2 H = \mu\sigma \frac{dH}{dt} + \mu\epsilon \frac{d^2H}{dt^2} \longrightarrow (8)$$

→ equations (7) & (8) are comes under conducting medium.

→ In dielectric mediums, conductivity $\sigma = 0$, then

$$(7) \Rightarrow \nabla^2 E = \mu\epsilon \frac{d^2E}{dt^2} \longrightarrow (9)$$

$$(8) \Rightarrow \nabla^2 H = \mu\epsilon \frac{d^2H}{dt^2} \longrightarrow (10)$$

→ equations (9) & (10) are comes under dielectric medium.

Let us the electromagnetic wave is propagating towards the Positive z-direction, then

$$(9) \Rightarrow \quad \nabla^2 E = \frac{\partial^2 E}{\partial z^2} = \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 E}{\partial t^2} = \frac{1}{\mu\epsilon}\frac{\partial^2 E}{\partial z^2}$$

$$\Rightarrow \frac{\partial^2 E}{\partial t^2} - \frac{1}{\mu\epsilon}\frac{\partial^2 E}{\partial z^2} = 0$$

$$\Rightarrow \boxed{\frac{\partial^2 E}{\partial t^2} - u^2\frac{\partial^2 E}{\partial z^2} = 0} \longrightarrow (11)$$

$$u = \frac{1}{\sqrt{\epsilon\mu}}$$

Similarly

$$(10) \Rightarrow \quad \nabla^2 H = \frac{\partial^2 H}{\partial z^2} = \mu\epsilon \frac{\partial^2 H}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 H}{\partial t^2} = \frac{1}{\mu\epsilon}\frac{\partial^2 H}{\partial z^2}$$

$$\Rightarrow \frac{\partial^2 H}{\partial t^2} - \frac{1}{\mu\epsilon}\frac{\partial^2 H}{\partial z^2} = 0$$

$$\Rightarrow \boxed{\frac{\partial^2 H}{\partial t^2} - u^2\frac{\partial^2 H}{\partial z^2} = 0} \longrightarrow (12)$$

→ The above 2nd order Partial differential equations (11) & (12) are called "scalar wave equations." where u is called velocity of electromagnetic wave.

$$u^2 = \frac{1}{\mu\epsilon} \Rightarrow u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_r\mu_0 \cdot \epsilon_r\epsilon_0}}$$

$$\Rightarrow u = \frac{1/\sqrt{\mu_0\epsilon_0}}{\sqrt{\mu_r\epsilon_r}} = \frac{c}{\sqrt{\mu_r\epsilon_r}} \Rightarrow \boxed{u = \frac{c}{\sqrt{\mu_r\epsilon_r}}}$$

→ In free space, $\mu_r\epsilon_r = 1$, then

$$\boxed{\begin{aligned} u = c &= 3\times10^8 \text{ m/sec} \\ &= 3,00,000 \text{ km/sec} \end{aligned}}$$

Helmholtz's Equations:

Helmholtz's equations (or) vector form of wave equations can be obtained from maxwell's equations in exponential form (or) phasor form.

→ The displacement with time can be expressed as

$$\vec{D} = De^{J\omega t} \quad \text{and} \quad \vec{B} = Be^{J\omega t}$$

where D & B are magnetudes of displacements and

$\omega = 2\pi f$ rad/sec angular frequency

then $$\frac{\partial \bar{D}}{\partial t} = J\omega . De^{J\omega t} = J\omega \bar{D}$$

and $$\frac{\partial \bar{B}}{\partial t} = J\omega . Be^{J\omega t} = J\omega \bar{B}$$

→ Therefore in maxwell's equations $\frac{\partial}{\partial t}$ is replaced with $J\omega$.

$$\nabla . D = \rho_v \Rightarrow \nabla . E = \frac{\rho_v}{\epsilon} \Rightarrow \nabla . E = 0 \longrightarrow (1)$$

$$\nabla . B = 0 \Rightarrow \nabla . \mu H = 0 \Rightarrow \nabla . H = 0 \longrightarrow (2)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \Rightarrow \nabla \times E = -J\omega\mu H \longrightarrow (3)$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \Rightarrow \nabla \times H = [\sigma + J\omega\epsilon]E \longrightarrow (4)$$

→ Apply curl operation to eq (3)

$$\nabla \times \nabla \times E = \nabla \times (-J\omega\mu H)$$

$$\nabla(\nabla . E) - \nabla^2 E = -J\omega\mu(\nabla \times H)$$

$$0 - \nabla^2 E = -J\omega\mu(\sigma + J\omega\epsilon)E \qquad \because \text{from eq(4).}$$

$$\nabla^2 E = J\omega\mu(\sigma + J\omega\epsilon)E$$

$$\nabla^2 E - J\omega\mu(\sigma + J\omega\epsilon)E = 0$$

$$\boxed{\therefore \nabla^2 E - \gamma^2 E = 0} \quad (\text{or}) \quad \boxed{\frac{\partial^2 E}{\partial z^2} - \gamma^2 E = 0} \longrightarrow (5).$$

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→ Apply curl operation to eq (4)

$$\nabla \times \nabla \times H = \nabla \times (\sigma + J\omega\epsilon) E$$

$$\nabla(\nabla \cdot H) - \nabla^2 H = (\sigma + J\omega\epsilon)(\nabla \times E)$$

$$0 - \nabla^2 H = (\sigma + J\omega\epsilon)(-J\omega\mu H) \quad \because \text{from eq (3)}.$$

$$-\nabla^2 H = -J\omega\mu H (\sigma + J\omega\epsilon)$$

$$\nabla^2 H - J\omega\mu H(\sigma + J\omega\epsilon) = 0$$

$$\therefore \boxed{\nabla^2 H - \gamma^2 H = 0} \quad (\text{or}) \quad \boxed{\frac{\partial^2 H}{\partial z^2} - \gamma^2 H = 0} \longrightarrow (6)$$

→ equations (5) & (6) are called vector form of wave equations (or) "Helmholtz's" wave equations. where 'γ' is called propagation time constant and it is measured in "m^{-1}".

$$\gamma^2 = J\omega\mu(\sigma + J\omega\epsilon)$$

$$\boxed{\gamma = \sqrt{J\omega\mu(\sigma + J\omega\epsilon)}}$$

→ The propagation constant γ is the complex quantity made up of real and imaginary terms. Thus,

$$\gamma = \alpha + J\beta = \sqrt{J\omega\mu(\sigma + J\omega\epsilon)}$$

where α → Attenuation constant (or) Attenuation factor
β → phase constant (or) wave number.

Attenuation constant : (α).

When a wave travels through medium it gets attenuated. That means the amplitude of the wave reduces. It is represented by the real part of propagation constant. It is called attenuation constant (α).

→ It is measured in Neper/meter (or) decibel (dB)

$\boxed{1 \text{ neper} = 8.686 \text{ dB}}$ (or) $\boxed{1 \text{ dB} = 0.115 \text{ nepers.}}$

Phase Constant : (β).

When a wave travels through the medium, phase change occurs. Such a phase change is expressed by an imaginary part of the propagation constant (γ). It is called phase shift constant (or) phase constant (β).

→ It is measured in radian/meter.

Derivation for α & β :

→ We know that propagation constant of Electromagnetic wave

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$$

$$\Rightarrow (\alpha + j\beta)^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

$$\alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu\sigma - \omega^2\mu\epsilon$$

⟹ Compare real parts on both sides.

$$\alpha^2 - \beta^2 = -\omega^2\mu\epsilon \Rightarrow \boxed{\beta^2 - \alpha^2 = \omega^2\mu\epsilon} \longrightarrow (a)$$

→ Take magnetude of $\alpha + j\beta$

$$|\alpha + j\beta| = \sqrt{\alpha^2 + \beta^2} = \left|\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}\right|$$

$$\sqrt{\alpha^2 + \beta^2} = \sqrt{\omega\mu\sqrt{(\sigma^2 + \omega^2\epsilon^2)}}$$

Squaring on both sides

$$\alpha^2 + \beta^2 = \omega\mu\sqrt{(\sigma^2 + \omega^2\epsilon^2)}$$

$$\alpha^2 + \beta^2 = \omega\mu\sqrt{\left[\left(\frac{\sigma}{\omega\epsilon}\right)^2 + 1\right]\omega^2\epsilon^2}$$

$$\boxed{\alpha^2 + \beta^2 = \omega^2\mu\epsilon\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}} \longrightarrow (b)$$

→ Subtract eq (a) from eq (b)

$$(b) - (a) \Rightarrow \alpha^2 + \beta^2 - \beta^2 + \alpha^2 = \omega^2\mu\epsilon\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - \omega^2\mu\epsilon$$

$$2\alpha^2 = \omega^2\mu\epsilon\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]$$

$$\alpha^2 = \frac{\omega^2\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]$$

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]}}$$

→ Addition of eq (a) and (b)

$$(a)+(b) \Rightarrow \beta^2-\alpha^2+\alpha^2+\beta^2 = \omega^2\mu\epsilon + \omega^2\mu\epsilon\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}$$

$$2\beta^2 = \omega^2\mu\epsilon\left[1+\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}\right]$$

$$\beta^2 = \frac{\omega^2\mu\epsilon}{2}\left[1+\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}\right]$$

$$\boxed{\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]}}$$

Solution of wave equation :

→ By solving the Helmholtz's wave equation (or) vector form of wave equations $\Rightarrow \nabla^2 E - \gamma^2 E = 0$ (or) $\frac{d^2E}{dz^2} - \gamma^2 E = 0$ and

$\Rightarrow \nabla^2 H - \gamma^2 H = 0$ (or) $\frac{d^2H}{dz^2} - \gamma^2 H = 0$

we get a electric field and magnetic field vectors, these vectors are the functions of both distance (z) and instantaneous time (t).

$$E(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\, a_x \longrightarrow (1)$$

$$H(z,t) = H_0 e^{-\alpha z}\cos(\omega t - \beta z)\, a_y \longrightarrow (2)$$

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Here

$E_0 \rightarrow$ Maximum amplitude of E wave

$H_0 \rightarrow$ Maximum amplitude of H wave

$\alpha \rightarrow$ Loss of propagation (or) Attenuation constant

$\beta \rightarrow$ Phase constant.

$\rightarrow$ If the Attenuation α is negligible then $e^{-\alpha z} = 1$,

$$eq(1) \Rightarrow E(z,t) = E_0 \cos(\omega t - \beta z)\, a_x \longrightarrow (3)$$

$$eq(2) \Rightarrow H(z,t) = H_0 \cos(\omega t - \beta z)\, a_y \longrightarrow (4)$$

$\rightarrow$ The above equations (3) and (4) are called time harmonic electro-magnetic fields.

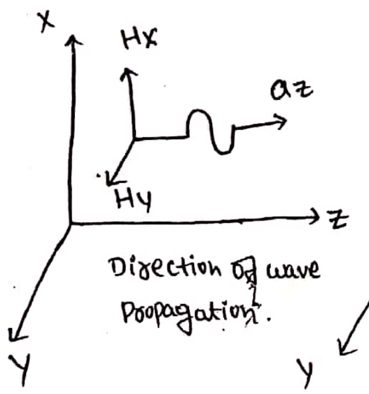
Uniform plane wave :

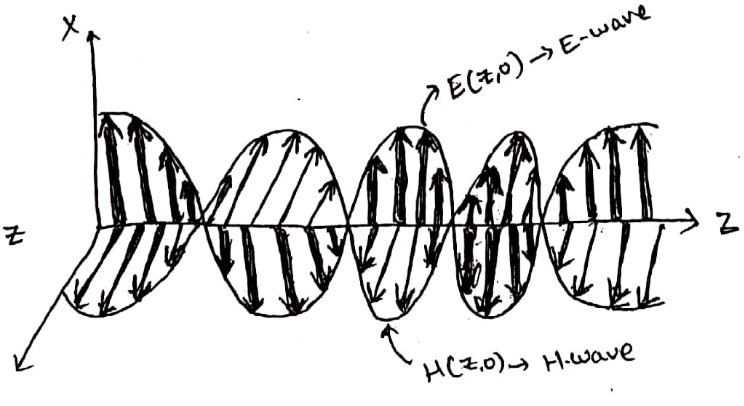
Electromagnetic wave (EM wave) Propagation is always orthogonal (or) transverse to the direction of E-wave and H-wave is called Transverse Electromagnetic (TEM) wave.

If the E-wave and H-wave has the same magnetude throughout the any transverse plane, then the resultant wave is called uniform plane wave.

$$E(z,t) = E_0 \cos(\omega t - \beta z)\, a_x \Rightarrow E(z,0) = E_0 \cos(-\beta z)\, a_x$$

$$H(z,t) = H_0 \cos(\omega t - \beta z)\, a_y \Rightarrow H(z,0) = H_0 \cos(-\beta z)\, a_y$$





Consider an EM wave propagating through the free space ($\sigma=0$). Consider that the electric field in the wave is in x-direction only while the magnetic field is in y-direction only. Both the fields i.e., E-field and H-field do not vary with x & y but vary only with z.

→ The E-field is in a_x direction while H-field is in a_y direction. That means E & H are lie in x-y plane. So in any of the planes in the wave, the vectors E & H are independent of x & y. Thus the E & H are the functions of z and t only.

→ The uniform plane wave is perpendicular to the plane consisting the E & H field vectors calleds the Transverse electromagnetic waves.

→ consider the wave equation for E-field

$$\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\epsilon \frac{\partial^2 E}{\partial t^2} \Rightarrow \nabla^2 E = \mu\epsilon \frac{\partial^2 E}{\partial t^2} \quad \Big| \text{in free space; } \sigma = 0$$

$$\Rightarrow \nabla^2 E = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \mu_0\epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

→ But the wave travel in the z-direction, hence E is independent of x & y. Hence first two diff terms in the above equation are zero. Hence we can write,

$$\frac{\partial^2 E}{\partial z^2} = \mu_0\epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

→ For the wave propagation in z-direction, E may have E_x and E_y components but definitely not E_z. According to assumption, E is in a_x direction, so let us consider that only E_x present. Then we can rewrite above equation as

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu_0\epsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

Similarly

$$\boxed{\frac{\partial^2 H_y}{\partial z^2} = \mu_0\epsilon_0 \frac{\partial^2 H_y}{\partial t^2}}$$

→ Therefore wave is propagatin in z-direction and E-field is in x-direction then H-field is in y-direction

Intrinsic Impedance (η) (or) Relation b/w E & H

The impedance offered by the medium is called intrinsic impedance. It is represented by η. It can be computed from the formula

$$\boxed{\eta = \frac{E_0}{H_0} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}}$$

Proof: From Maxwell's equations,

$$\nabla \times E = -\frac{\partial B}{\partial t} = -j\omega\mu H$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} = (\sigma + j\omega\epsilon) E$$

and solutions of wave equations,

$$E(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_x$$

$$H(z,t) = H_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_y$$

$$\Rightarrow \quad \nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E(z,t) & 0 & 0 \end{vmatrix}$$

$$\nabla \times E = a_x[0-0] - a_y\left[0 - \frac{\partial}{\partial z} E(z,t)\right] + a_z\left[0 - \frac{\partial}{\partial y} E(z,t)\right]$$

$$\nabla \times E = a_x(0) + a_y \frac{\partial}{\partial z}\left[E_0 e^{-\alpha z} \cos(\omega t - \beta z)\right] + a_z(0)$$

$$\nabla \times E = E_0 \frac{\partial}{\partial z}\left[e^{-\alpha z} \cos(\omega t - \beta z)\right] a_y \longrightarrow (1)$$

→ from maxwell's equation

$$\nabla \times E = -j\omega\mu H = -j\omega\mu H_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_y \longrightarrow (2)$$

→ equating eq (1) & (2)

$$-j\omega\mu H_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \cancel{a_y} = E_0 \frac{\partial}{\partial z}\left[e^{-\alpha z} \cos(\omega t - \beta z)\right] \cancel{a_y}$$

$$\therefore \; -j\omega\mu H_0 e^{-\alpha z} \cos(\omega t - \beta z) = E_0 \frac{\partial}{\partial z}\left[e^{-\alpha z} \cos(\omega t - \beta z)\right] \longrightarrow (3).$$

$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & H(z,t) & 0 \end{vmatrix}$$ 4.6

$$\nabla \times H = a_x\left[0 - \frac{d}{dz}H(z,t)\right] - a_y[0-0] + a_z\left[\frac{d}{dx}H(z,t) - 0\right]$$

$$\nabla \times H = -a_x \frac{d}{dz}\left[H_0 e^{-\alpha z}\cos(\omega t - \beta z)\right] - a_y(0) + a_z(0)$$

$$\nabla \times H = -H_0 \frac{d}{dz}\left[e^{-\alpha z}\cos(\omega t - \beta z)\right] a_x \longrightarrow (4)$$

→ from Maxwell's equation

$$\nabla \times H = (\sigma + J\omega\epsilon)E = (\sigma + J\omega\epsilon)E_0 e^{-\alpha z}\cos(\omega t - \beta z)\, a_x \longrightarrow (5)$$

→ equating eq (4) & (5)

$$(\sigma + J\omega\epsilon)E_0 e^{-\alpha z}\cos(\omega t - \beta z)\cancel{a_x} = -H_0 \frac{d}{dz}\left[e^{-\alpha z}\cos(\omega t - \beta z)\right]\cancel{a_x}$$

$$\therefore (\sigma + J\omega\epsilon)E_0 e^{-\alpha z}\cos(\omega t - \beta z) = -H_0 \frac{d}{dz}\left[e^{-\alpha z}\cos(\omega t - \beta z)\right] \longrightarrow (6)$$

→ dividing eq (3) with (6)

$$\Rightarrow \frac{-J\omega\mu H_0 e^{-\alpha z}\cos(\omega t - \beta z)}{(\sigma + J\omega\epsilon)E_0 e^{-\alpha z}\cos(\omega t - \beta z)} = \frac{E_0 \frac{d}{dz}\left[e^{-\alpha z}\cos(\omega t - \beta z)\right]}{-H_0 \frac{d}{dz}\left[e^{-\alpha z}\cos(\omega t - \beta z)\right]}$$

$$\Rightarrow \frac{-J\omega\mu H_0}{(\sigma + J\omega\epsilon)E_0} = \frac{-E_0}{H_0}$$

$$\Rightarrow \frac{J\omega\mu}{(\sigma + J\omega\epsilon)} = \frac{E_0^2}{H_0^2}$$

$$\Rightarrow \frac{E_0}{H_0} = \sqrt{\frac{J\omega\mu}{(\sigma + J\omega\epsilon)}}$$

$$\boxed{\therefore \eta = \frac{E_0}{H_0} = \sqrt{\frac{J\omega\mu}{(\sigma + J\omega\epsilon)}}}, \quad \boxed{\eta = |\eta| \angle \theta_n}, \quad \boxed{H_0 = \frac{E_0}{\eta}}$$

Here $|\eta| \to$ magnitude of intrinsic impedance

$\theta_n \to$ phase of intrinsic impedance.

Velocity of EM wave (v):

It is defined as the velocity of propagation of the wave

$$v = \frac{dz}{dt} = \frac{\omega}{\beta} \qquad \therefore \boxed{v = \frac{\omega}{\beta}} \text{ m/sec}$$

Here $\omega = 2\pi f \rightarrow$ angular frequency

$\beta \rightarrow$ phase constant rad/m

In general $\rightarrow$ $\boxed{\beta = \omega\sqrt{\mu\epsilon}}$

Wave length of EM wave (λ):

It is defined as the distance through which the Sinusoidal wave passes through a full cycle of 2π radians,

i.e, $\boxed{\lambda = \frac{2\pi}{\beta}}$ meters

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Wave Propagation in lossy Dielectrics :

A lossy dielectric is a medium in which an EM wave loses Power as it propagates due to Poor conduction. In other words, a lossy dielectric is a Partially conducting medium i.e imperfect dielectric (or) imperfect conductor.

→ A lossy dielectric is a medium in which conductivity $\sigma \neq 0$, Permitivity $\epsilon = \epsilon_0 \epsilon_r$ $(\epsilon_r \neq 1)$ and Permeability $\mu = \mu_0 \mu_r$ $(\mu_r \neq 1)$.

a). Wave equations:

$$\nabla^2 E - \gamma^2 E = 0 \quad (\text{or}) \quad \frac{\partial^2 E}{\partial z^2} - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0 \quad (\text{or}) \quad \frac{\partial^2 H}{\partial z^2} - \gamma^2 H = 0$$

b) Propagation constant:

$$\gamma = \sqrt{J\omega\mu(\sigma + J\omega\epsilon)} = \alpha + J\beta$$

Attenuation constant:

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

phase constant:

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$$

c) Intrinsic impedance:

$$\eta = \sqrt{\frac{J\omega\mu}{\sigma + J\omega\epsilon}} = |\eta| \angle\theta_\eta$$

$$\eta = \sqrt{\frac{J\omega\mu}{J\omega\epsilon\left(\frac{\sigma}{J\omega\epsilon}+1\right)}}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon\left(1+\frac{\sigma}{J\omega\epsilon}\right)}}$$

$$\eta = \frac{\sqrt{\mu/\epsilon}}{\sqrt{1 - j\frac{\sigma}{\omega\epsilon}}}$$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}} = \frac{\sqrt{\mu/\epsilon}}{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}},$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon} \Rightarrow 2\theta_\eta = \tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)$$

$$\Rightarrow \boxed{\theta_\eta = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)}$$

d) Solutions of wave equations:

$$E(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_x$$

$$H(z,t) = H_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_y$$

→ But we know that relation b/w E_0 & H_0 is $\eta = \frac{E_0}{H_0} \Rightarrow H_0 = \frac{E_0}{\eta} = \frac{E_0}{|\eta| \angle \theta_\eta}$

then $H(z,t) = \frac{E_0}{|\eta|\angle\theta_\eta} e^{-\alpha z} \cos(\omega t - \beta z)\, a_y = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, a_y$

→ E-field leads H-field by θ_η (or) H-field lags E-field by θ_η.

e) Loss tangent (or) loss angle:

It is the measure of lossy medium. It can be defined as the ratio of magnetude of conduction current density to the displacement current density and it is denoted with $\tan\theta$.

$$\tan\theta = \left|\frac{J_c}{J_D}\right| = \left|\frac{\sigma E}{\partial D/\partial t}\right| = \left|\frac{\sigma E}{j\omega\epsilon E}\right| = \left|\frac{\sigma}{j\omega\epsilon}\right| \Rightarrow$$

$$\boxed{\tan\theta = \frac{\sigma}{\omega\epsilon}} \text{ (or) } \boxed{\theta = \tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)}$$

$\theta \rightarrow$ loss angle

$\tan\theta \rightarrow$ loss tangent

→ Relation b/w θ_η & θ:

$$\theta_\eta = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)$$

$$\theta_\eta = \frac{1}{2}\theta \Rightarrow \boxed{\theta = 2\theta_\eta}$$

f) Velocity of EM wave:

$$\boxed{u = \frac{\omega}{\beta}} \text{ meter/sec}$$

v

g) wavelength of EM wave:

$$\boxed{\lambda = \frac{2\pi}{\beta}} \text{ meters}$$

Wave Propagation in Lossless (or) Perfect Dielectrics :

Lossless dielectric is a medium in which conductivity $\sigma = 0$, Permitivity $\epsilon = \epsilon_0 \epsilon_r$, $\mu = \mu_0 \mu_r$ and loss tangent $\frac{\sigma}{\omega\epsilon} << 1$.

a) Wave equations :

$$\nabla^2 E - \gamma^2 E = 0 \quad (\text{or}) \quad \frac{d^2 E}{dz^2} - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0 \quad (\text{or}) \quad \frac{d^2 H}{dz^2} - \gamma^2 H = 0.$$

b) Propagation constant :

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

If $\sigma = 0$, then $\gamma = \sqrt{j\omega\mu(j\omega\epsilon)} = j\omega\sqrt{\mu\epsilon}$

→ This propagation constant having only imaginary term no real term it means $\alpha = 0$ and $\beta = \omega\sqrt{\mu\epsilon}$.

Attenuation constant :

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1} = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1+0} - 1} = 0 \qquad \boxed{\therefore \alpha = 0}$$

Phase constant :

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1} = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1+0} + 1} = \omega\sqrt{\mu\epsilon}.$$

$$\boxed{\therefore \beta = \omega\sqrt{\mu\epsilon}}$$

c) Intrinsic Impedance :

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

If $\sigma = 0$, then $\eta = \sqrt{\frac{j\omega\mu}{0 + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\epsilon}} = \sqrt{\frac{\mu}{\epsilon}}$

$$\eta = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \sqrt{\frac{\mu_r}{\epsilon_r}} \times \sqrt{\frac{4\pi \times 10^{-7}}{\frac{1}{36\pi \times 10^9}}} = \sqrt{144\pi^2 \times 10^2} \times \sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$\eta = 120\pi\sqrt{\frac{\mu_r}{\epsilon_r}} = 377\sqrt{\frac{\mu_r}{\epsilon_r}} \ \Omega$$

$$\boxed{\therefore \eta = 377\sqrt{\frac{\mu_r}{\epsilon_r}}\ \Omega}$$ (or)

$$\boxed{\eta = \sqrt{\frac{\mu}{\epsilon}}\angle 0^\circ ;\quad \theta_\eta = 0^\circ}$$

d) Solutions of wave equations :

$$E(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\, a_x$$

If $\alpha = 0$, then $E(z,t) = E_0 \cos(\omega t - \beta z)\, a_x$.

$$H(z,t) = H_0 e^{-\alpha z}\cos(\omega t - \beta z)\, a_y$$

In this case $\alpha = 0$ & Generally $\eta = \frac{E_0}{H_0} \Rightarrow H_0 = \frac{E_0}{\eta}$ & $\eta = |\eta|\angle 0^\circ$

$$\therefore H(z,t) = \frac{E_0}{|\eta|}\cos(\omega t - \beta z)\, a_y.$$

→ That means E- field and H- fields are in phase with each other

e) Velocity of EM wave :

$$u = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}}$$

$\mu = \mu_r\mu_0$ & $\epsilon = \epsilon_r\epsilon_0$ then

$$u = \frac{1}{\sqrt{\mu_r\epsilon_r}\cdot\sqrt{\frac{4\pi\times10^{-7}}{36\pi\times10^{9}}}} = \frac{1}{\sqrt{\mu_r\epsilon_r}\sqrt{\frac{1}{9\times10^{16}}}}$$

$$u = \frac{\sqrt{9\times10^{16}}}{\sqrt{\mu_r\epsilon_r}} = \frac{3\times10^{8}}{\sqrt{\mu_r\epsilon_r}} = \frac{c}{\sqrt{\mu_r\epsilon_r}}$$

$\therefore c = 3\times10^{8}$ m/sec

$$\boxed{\therefore u = \frac{c}{\sqrt{\mu_r\epsilon_r}}} \text{ m/sec}$$

f) Wave length of EM wave :

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\epsilon}} = \frac{2\pi}{2\pi f\sqrt{\mu\epsilon}} = \frac{1}{f\sqrt{\mu\epsilon}}$$

$$\boxed{\therefore \lambda = \frac{1}{f\sqrt{\mu\epsilon}}}$$

Wave Propagation in free space :

Free space is a medium in which the conductivity $\sigma = 0$, permittivity $\epsilon = \epsilon_0$ ($\epsilon_r = 1$) and permeability $\mu = \mu_0$ ($\mu_r = 1$).

a) Wave equations :

$$\nabla^2 E - \gamma^2 E = 0 \quad (\text{or}) \quad \frac{d^2 E}{dz^2} - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0 \quad (\text{or}) \quad \frac{d^2 H}{dz^2} - \gamma^2 H = 0.$$

b) Propagation constant :

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} \quad ; \sigma = 0,\ \mu = \mu_0 \ \&\ \epsilon = \epsilon_0.$$

$$\gamma = \sqrt{j\omega\mu_0(0 + j\omega\epsilon_0)}$$

$$\gamma = \sqrt{j\omega\mu_0\, j\omega\epsilon_0} = j\omega\sqrt{\mu_0\epsilon_0}$$

→ The propagation constant having only imaginary term no real term.
It means $\alpha = 0$ & $\beta = \omega\sqrt{\mu_0\epsilon_0}$.

Attenuation constant :

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1} = \omega\sqrt{\frac{\mu_0\epsilon_0}{2}\sqrt{1+0} - 1} = 0. \quad \boxed{\therefore \alpha = 0}$$

Phase constant :

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1} = \omega\sqrt{\frac{\mu_0\epsilon_0}{2}\sqrt{1+0} + 1} = \omega\sqrt{\mu_0\epsilon_0} \quad \boxed{\therefore \beta = \omega\sqrt{\mu_0\epsilon_0}}$$

c) Intrinsic Impedance :

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \quad ; \sigma = 0,\ \mu = \mu_0 \ \&\ \epsilon = \epsilon_0$$

$$\eta = \sqrt{\frac{j\omega\mu_0}{j\omega\epsilon_0}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{4\pi\times10^{-7}\times36\pi\times10^{9}}$$

$$\eta = 2\times\pi\times6\times10 = 120\pi = 377\ \Omega$$

$$\boxed{\therefore \eta = 120\pi = 377\ \Omega}$$

$$\boxed{\therefore \theta_\eta = 0^\circ}$$

$\eta = |\eta| \angle\theta_\eta$

$\eta = 377\Omega\ \angle 0^\circ$

d) Solutions of wave equations :

$E(z,0) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_x$; $\alpha = 0$;

$E(z,0) = E_0 \cos(\omega t - \beta z)\, a_x$.

$H(z,0) = H_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_y$; $\alpha = 0$ & $\eta = \frac{E_0}{H_0} = 120\pi$

$\Rightarrow H_0 = \frac{E_0}{120\pi}$.

$H(z,0) = \frac{E_0}{120\pi} \cos(\omega t - \beta z)\, a_y$.

and $\eta = |\eta| \angle 0^\circ$

→ therefore E-field & H-fields are in phase with each other

e) Velocity of EM-wave :

$$u = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$u = \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{1}{36\pi \times 10^{9}}}} = \sqrt{9 \times 10^{16}} = 3 \times 10^{8} \text{ m/sec} = c.$$

$$\boxed{\therefore u = c = 3 \times 10^{8} \text{ m/sec} = 3,00,000 \text{ km/sec}}$$

f) Wavelength of EM wave :

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu_0 \epsilon_0}} = \frac{2\pi}{2\pi f \sqrt{\mu_0 \epsilon_0}} = \frac{1}{f\sqrt{\mu_0 \epsilon_0}}$$

$\because c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$$\lambda = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \cdot \frac{1}{f} = \frac{c}{f} \Rightarrow \boxed{\lambda = \frac{c}{f}} \Rightarrow \boxed{c = f\lambda}$$

Wave propagation in Good Conductors :

A Good conductor is a medium (or) media in which the conductivity $\sigma = \infty$. Permitivity $\epsilon = \epsilon_0$ ($\epsilon_r = 1$), Permeability $\mu = \mu_0 \mu_r$ ($\mu_r \neq 1$). and the loss tangent $\frac{\sigma}{\omega\epsilon} >> 1$.

a) **Wave equations :**

$$\nabla^2 E - \gamma^2 E = 0 \quad \text{(or)} \quad \frac{\partial^2 E}{\partial z^2} - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0 \quad \text{(or)} \quad \frac{\partial^2 H}{\partial z^2} - \gamma^2 H = 0.$$

b) **Propagation constant :**

$$\gamma = \sqrt{J\omega\mu(\sigma + J\omega\epsilon)} \quad : \sigma = \infty \text{ then neglect } J\omega\epsilon;$$

$$\gamma = \sqrt{J\omega\mu\sigma}$$

$$\gamma = \sqrt{J}\sqrt{\omega\mu\sigma}$$

$$\gamma = \frac{1+J}{\sqrt{2}}\sqrt{2\pi f\mu\sigma}$$

$$\gamma = \sqrt{\pi f\mu\sigma} + J\sqrt{\pi f\mu\sigma} = \alpha + J\beta$$

$$J = 0 + J\cdot 1 = \cos\frac{\pi}{2} + J\sin\frac{\pi}{2}$$

$$\sqrt{J} = \sqrt{\cos\frac{\pi}{2} + J\sin\frac{\pi}{2}}$$

$$= \left(\cos\frac{\pi}{2} + J\sin\frac{\pi}{2}\right)^{1/2}$$

$$= \cos\frac{\pi}{4} + J\sin\frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}} + \frac{J}{\sqrt{2}}$$

$$\sqrt{J} = \frac{1+J}{\sqrt{2}}$$

$$\therefore \alpha = \sqrt{\pi f\mu\sigma} \quad \& \quad \beta = \sqrt{\pi f\mu\sigma}$$

$$\therefore \gamma = |\gamma|\angle\theta = \sqrt{\omega\mu\sigma}\angle\pi/4 = \sqrt{\omega\mu\sigma}\angle 45^\circ$$

Attenuation constant :

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]} = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\frac{\sigma}{\omega\epsilon}\right)} = \omega\sqrt{\frac{\mu\sigma}{2\omega}} = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\therefore \alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$$

Phase constant :

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]} = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\frac{\sigma}{\omega\epsilon}\right)} = \omega\sqrt{\frac{\mu\sigma}{2\omega}} = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma} \qquad \Big| \; \frac{\sigma}{\omega\epsilon} >> 1$$

c) Intrinsic impedance :

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$\eta = \sqrt{j}\sqrt{\frac{\omega\mu}{\sigma}} = \left(\frac{1+j}{\sqrt{2}}\right)\sqrt{\frac{\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$$

$$\therefore \eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle \pi/4$$

$$\boxed{\therefore \theta_\eta = \frac{\pi}{4} = 45^\circ} \qquad \boxed{\eta = \sqrt{\frac{2\pi f\mu}{2\sigma}} = \sqrt{\frac{\pi f\mu}{\sigma}}}$$

d) Solutions of wave equations :

$$E(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\, a_x$$

$$H(z,t) = H_0 e^{-\alpha z}\cos(\omega t - \beta z)\, a_y = \frac{E_0}{|\eta|\angle\pi/4} e^{-\alpha z}\cos(\omega t - \beta z)\, a_y$$

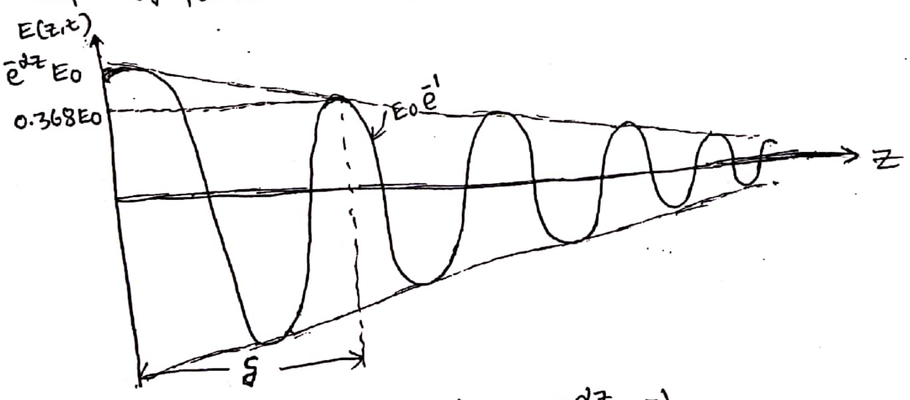
$$\eta = \frac{E_0}{H_0}$$

$$\eta = |\eta|\angle\pi/4$$

→ There fore E-field Leads H-field by 45° (or) H-field lags E-field by 45°.

Depth of Penetration (or) Skin depth :

When an electromagnetic wave is propagating through a conducting medium, then its amplitude is attenuated by a factor $e^{-\alpha z}$. The distance travelled by the wave with in the Conductor, for an attenuation of "e^{-1}" (or) 37% of original amplitude is called skin depth (or) depth of penetration. It is denoted with δ and measured in meter.



→ If $z = \delta$, then $E_0 e^{-\alpha z} = E_0 e^{-1} \Rightarrow e^{-\alpha z} = e^{-1} \Rightarrow \alpha z = 1 \Rightarrow z = \frac{1}{\alpha}$

$$\therefore \delta = z = \frac{1}{\alpha} = \frac{1}{\beta} = \sqrt{\frac{2}{\omega\mu\sigma}} \qquad \boxed{\therefore \delta = \sqrt{\frac{2}{\omega\mu\sigma}}}$$

Polarisation :

Def 1 : The time varying behaviour of an electric field strength vector at some fixed point in space is called Polarisation.

Def 2 : The Polarisation of a wave is defined as the direction of electric-field at a given point as a function of time.

Def 3 : The orientation of electric field is called Polarisation.

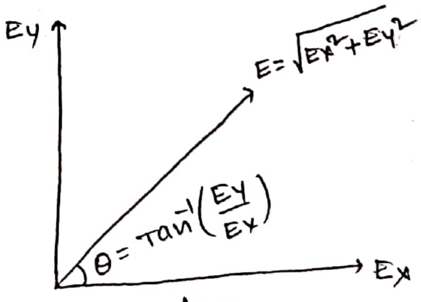
→ There are three types of Polarisations, namely
a) Linear Polarisation
b) Circular Polarisation
c) Elliptical Polarisation.

a) Linear Polarisation :

A wave is said to be Linearly Polarised if the electric field remains along a straight line as a function of time at some point in the medium.

→ Linear polarisation of wave is again of three types, namely

(i) Horizontal polarisation
(ii) Vertical Polarisation
(iii) Theta (θ) Polarisation.



fig(1).

→ When a wave travels in z-direction with E and H fields lying in XY-plane, if $E_y = 0$ and E_x is present, it is said to be X-Polarised (or) horizontally Polarised.

→ If $E_x = 0$ and E_y is only present, the wave is said to be Y-Polarised (or) vertically polarised.

→ If E_x and E_y are present and are in inphase, then the wave is said to be theta (θ) Polarised.

Explanation : If $E = E_x a_x + E_y a_y$ and E_x, E_y are in inphase. Then the resultant wave propagates with relative magnitude $\sqrt{E_x^2 + E_y^2}$ which makes an angle of $\theta = Tan^{-1}\left(\frac{E_y}{E_x}\right)$ with Positive x-axis is called Linear Polarisation as shown in fig(1).

b) Circular Polarisation :

If E_x and E_y are in equal magnitude and a phase difference of $90°$, then the locus of the resultant electric field vector E is circular and the wave is said to be circularly polarised.

$$E = E_a \cos\omega t \, a_x + E_a \cos(\omega t + \tfrac{\pi}{2}) a_y = E_a \cos\omega t \, a_x - E_a \sin\omega t \, a_y$$

Compare it with $E = E_x a_x + E_y a_y$ then

$$E_x = E_a \cos\omega t \quad \& \quad E_y = -E_a \sin\omega t$$

$$\therefore E_x^2 + E_y^2 = E_a^2 \cos^2\omega t + E_a^2 \sin^2\omega t = E_a^2 (1).$$

$$\boxed{\therefore E_x^2 + E_y^2 = E_a^2}$$

c) Elliptical Polarisation :

If E_x and E_y are in different magnitudes, and a phase-difference of $90°$, then the locus of the resultant electric field vector E is an ellipse and the wave is said to be elliptically polarised.

$$E = E_a \cos\omega t \, a_x + E_b \cos(\omega t + \tfrac{\pi}{2}) a_y$$

$$E = E_a \cos\omega t \, a_x - E_b \sin\omega t \, a_y$$

Comparing it with $E = E_x a_x + E_y a_y$

$$E_x = E_a \cos\omega t \quad \& \quad E_y = -E_b \sin\omega t$$

$$\frac{E_x}{E_a} = \cos\omega t \quad \& \quad \frac{E_y}{E_b} = -\sin\omega t$$

$$\left(\frac{E_x}{E_a}\right)^2 + \left(\frac{E_y}{E_b}\right)^2 = \cos^2\omega t + \sin^2\omega t = 1.$$

$$\boxed{\therefore \left(\frac{E_x}{E_a}\right)^2 + \left(\frac{E_y}{E_b}\right)^2 = 1}$$

① Find α, β, γ and η for ferrite at 10 GHz, $\epsilon_r = 9$, $\mu_r = 4$ and $\sigma = 10$ m S/m.

Sol Given data $f = 10\text{ GHz} = 10 \times 10^9\text{ Hz}$, $\epsilon_r = 9$, $\mu_r = 4$

$\sigma = 10\text{ m S/m} = 10 \times 10^{-3}\text{ S/m}$.

→ We need to determine the loss tangent to be able to tell the medium.

$$\operatorname{Tan}\theta = \frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2\pi f \epsilon_0 \epsilon_r} = \frac{10 \times 10^{-3}}{2\pi \times 10 \times 10^9 \times \frac{1}{36\pi \times 10^9}} \times 9 = 2 \times 10^{-3} = 0.002$$

$\boxed{\therefore \frac{\sigma}{\omega\epsilon} << 1}$ Therefore it is a dielectric media (or) Perfect dielectric media.

(a). $$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]} = 2\pi f \sqrt{\frac{\mu_r \mu_0 \epsilon_r \epsilon_0}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]} \simeq 0$$

The value of α is 0.000002 Np/m. It is negligible. Approximately equal to zero.

(b). $$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]} = 400\text{ rad/m}$$

If $\beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\epsilon_0}.\sqrt{\mu_r\epsilon_r} = \frac{\omega}{c}\sqrt{\mu_r\epsilon_r} = \frac{2\pi f \times \sqrt{\mu_r\epsilon_r}}{c}$

$$\beta = \frac{2\pi \times 10^{10}}{3 \times 10^8}\sqrt{9 \times 4} = \frac{2\pi}{3} \times 10^2 \times 6 = 1256.6\text{ rad/m}$$

(c) $\gamma = \alpha + j\beta = 0 + j\,400$
$= j\,400\ m^{-1}$

| $\gamma = 0 + j\,1256.6$
$= j\,1256.6\ m^{-1}$

(d). $$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \qquad \frac{\sigma}{\omega\epsilon} << 1$$

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\epsilon}} = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0} \times \frac{\mu_r}{\epsilon_r}}$$

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \times \sqrt{\frac{\mu_r}{\epsilon_r}} = 120\pi\sqrt{\frac{4}{9}} = \cancel{120}^{40}\pi \times \frac{2}{\cancel{3}} = 80\pi$$

$\therefore \eta = 80\pi = 251.32\ \Omega$.

Problem (2) A large copper conductor ($\sigma = 5.8\times10^7$ S/m, $\epsilon_r = \mu_r = 1$) support a uniform plane wave at 60 Hz. Determine

a) The ratio of conduction current to displacement current.

b) Attenuation, phase, and propagation constants.

c) Intrinsic impedance

d) wave length and phase velocity of propagation.

Sol Given data $\sigma = 5.8\times10^7$ semen/meter, $\epsilon_r = \mu_r = 1$ & $f = 60$ Hz

loss tangent $\frac{\sigma}{\omega\epsilon} = \frac{5.8\times10^7}{2\pi\times60\times\frac{1}{36\pi\times10^9}\times1} = \frac{5.8\times10^{16}\times18}{60} = 1.74\times10^{16}$ $\boxed{\therefore \frac{\sigma}{\omega\epsilon} >> 1}$

a) $\frac{J_c}{J_D} = \frac{\sigma E}{\frac{\partial D}{\partial t}} = \frac{\sigma E}{j\omega\epsilon E} = \frac{\sigma}{j\omega\epsilon} = \frac{-j\sigma}{\omega\epsilon} = -j\,1.74\times10^{16}$

b) $\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1} = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{\left(\frac{\sigma}{\omega\epsilon}\right)^2}} = \omega\sqrt{\frac{\mu\epsilon}{2}\frac{\sigma}{\omega\epsilon}} = \omega\sqrt{\frac{\mu\sigma}{2\omega}}$

$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi f\times\mu_0\mu_r\times\sigma}{2}} = \sqrt{\frac{2\pi\times60\times4\pi\times10^{-7}\times5.8\times10^7}{2}} = 117.21$ NP/m

$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1} = \omega\sqrt{\frac{\mu\sigma}{2\omega}} = \sqrt{\frac{\omega\mu\sigma}{2}} = 117.21$ rad/m.

$\gamma = \alpha + j\beta = 117.21 + j\,117.21\ m^{-1}$.

c) $\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{j}\sqrt{\frac{\omega\mu}{\sigma}} = \left(\frac{1+j}{\sqrt2}\right)\sqrt{\frac{2\pi\times60\times4\pi\times10^{-7}\times1}{5.8\times10^7}}$

$\eta = \left(\frac{1+j}{\sqrt2}\right)(2.85\times10^{-6}) = (1+j)\,2.02\times10^{-6} = (2.02 + j\,2.02)\times10^{-6}\ \Omega$

d) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{117.21} = 0.0536$ m

$u = \frac{\omega}{\beta} = \frac{2\pi\times60}{117.21} = 3.22$ m/sec.

4.13

A lossy dielectric has an intrinsic impedance of $200 \angle 30^\circ\ \Omega$ a particular frequency. If at that frequency, the plane wave propagating through the dielectric has the magnetic field component $H = 10 e^{-\alpha x} \cos(\omega t + \frac{1}{2}x)\, a_y$ A/m. Find E and α. Determine the skin depth and wave polarisation.

Sol Given data $\eta = 200 \angle 30^\circ\ \Omega \Rightarrow |\eta| = 200\ \Omega$ & $\theta_\eta = 30^\circ = \frac{\pi}{6}$

$H = 10 e^{-\alpha x} \cos(\omega t - \frac{1}{2}x)\, a_y$ A/m $\Rightarrow H_0 = 10,\ \beta = \frac{1}{2}$

→ wave is along X-direction, H-field is along Y-direction then E-field is along z-direction.

$$\left.\begin{aligned} a_x \times a_y &= a_z \\ a_y \times a_z &= a_x \\ a_z \times a_x &= a_y \end{aligned}\right\} \quad \left.\begin{aligned} a_E \times a_H &= a_K \\ a_H \times a_K &= a_E \\ a_K \times a_E &= a_H \end{aligned}\right\}$$

Generally a_E is along x-direction
a_H is along y-direction
a_K is along z-direction

→ a_E, a_H & a_K are unit vectors along E-field, H-field & EM wave directions respectively

$a_x \times a_y = a_z \Rightarrow a_K \times a_H = -a_E \Rightarrow a_E = -a_z$ direction.

(a) E

$\eta = \frac{E_0}{H_0} = |\eta| \angle \theta_\eta \Rightarrow E_0 = H_0 |\eta| \angle \theta_\eta = 10 \times 200 \angle 30^\circ = 2000 \angle \frac{\pi}{6}$ V/m.

$E = E_0 e^{-\alpha x} \cos(\omega t - \frac{1}{2}x)(-a_z) = -2000 \angle \frac{\pi}{6}\, e^{-\alpha x} \cos(\omega t - \frac{1}{2}x)\, a_z$

$\therefore E = -2000\, e^{-\alpha x} \cos(\omega t - \frac{x}{2} + \frac{\pi}{6})\, a_x = -2\, e^{-\alpha x} \cos(\omega t - \frac{x}{2} + \frac{\pi}{6})\, a_z$ kV/m.

(b) α

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}\ ;\ \beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}\ ;\ \frac{\alpha}{\beta} = \frac{\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1}}{\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1}}$$

→ we know that $\theta_\eta = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right) \Rightarrow \frac{\sigma}{\omega\epsilon} = \tan(2\theta_\eta) = \tan(2\times 30) = \tan 60 = \sqrt{3}$

then $$\alpha = \beta \frac{\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1}}{\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1}} = \frac{1}{2}\frac{\sqrt{\sqrt{1+3} - 1}}{\sqrt{\sqrt{1+3} + 1}} = \frac{1}{2}\frac{\sqrt{2-1}}{\sqrt{2+1}} = \frac{1}{2} \times \frac{1}{\sqrt{3}} = 0.288\ \text{Np/m}$$

$\boxed{\therefore \alpha = 0.288\ \text{Np/m}}$

(c) δ

$\delta = \frac{1}{\alpha} = \frac{1}{0.288} = 3.472$ $\boxed{\therefore \delta = 3.472}$

(d) The wave has an E_z component; Hence it is polarised along the z-direction.

Problem (10.4) A uniform plane wave propagating in a medium has $E = 2e^{-\alpha z}\sin(10^8 t - \beta z)\,a_y$ V/m. If the medium is characterised by $\epsilon_r = 1$, $\mu_r = 20$ and $\sigma = 3$ ℧/m, find α, β & H.

Sol Given data: $E = 2e^{-\alpha z}\sin(10^8 t - \beta z)\,a_y$ V/m $\Rightarrow E_0 = 2,\ \omega = 10^8$

$\epsilon_r = 1,\ \mu_r = 20$ & $\sigma = 3$ ℧/m (or) Semen/meter.

→ We need to determine the loss tangent to be able to tell the medium.

$$\frac{\sigma}{\omega\epsilon} = \frac{3}{10^8 \times 1 \times \frac{1}{36\pi\times10^9}} = 3\times36\pi\times10 = 3393 \qquad \boxed{\therefore \frac{\sigma}{\omega\epsilon} >> 1}$$

→ Therefore the medium is a good conductor.

(a)
$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]} = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{\left(\frac{\sigma}{\omega\epsilon}\right)^2}} = \omega\sqrt{\frac{\mu\epsilon}{2}\frac{\sigma}{\omega\epsilon}} = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\alpha = \sqrt{\frac{\omega\mu_0\mu_r\sigma}{2}} = \sqrt{\frac{10^8\times4\pi\times10^{-7}\times20\times3}{2}} = 61.4 \text{ Np/m}. \qquad \boxed{\therefore \alpha = 61.4 \text{ Np/m}}$$

(b)
$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]} = \sqrt{\frac{\omega\mu\sigma}{2}} = 61.4 \text{ rad/m}. \qquad \boxed{\therefore \beta = 61.4 \text{ rad/m}}$$

(c)
$$\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}}\ ; \quad |\eta| = \sqrt{\frac{\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu_0\mu_r}{\sigma}}$$

$$|\eta| = \sqrt{\frac{10^8\times4\pi\times10^{-7}\times20}{3}} = \sqrt{\frac{800\pi}{3}} = 28.94\ \Omega$$

$$\tan 2\theta_n = \frac{\sigma}{\omega\epsilon} = 3393 \Rightarrow \theta_n = \frac{1}{2}\tan^{-1}(3393) = 45^\circ = \frac{\pi}{4}.$$

$$\boxed{\therefore \theta_n = \pi/4}$$

$$\therefore H = H_0 e^{-\alpha z}\sin\left(\omega t - \beta z - \frac{\pi}{4}\right) a_H$$

→ The wave is along z-direction, E-field is along y-direction then H-field is along x-direction.

$$a_z \times a_y = -a_x \Rightarrow a_K \times a_E = a_H \Rightarrow a_H = -a_x \text{ direction}$$

$$H_0 = \frac{E_0}{|\eta|} = \frac{2}{28.94} = 69.1\times10^{-3}$$

$$\therefore H = -69.1\, e^{-61.4z}\sin\left(10^8 t - 61.4z - \frac{\pi}{4}\right) a_x \text{ mA/m}.$$